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A new transient method for determining soil hydraulic conductivity function

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Abstract:

Instantaneous profile method (IPM) is a transient method for measuring a soil hydraulic conductivity function (SHCF), which relates soil hydraulic conductivity with suction. In the existing interpretation method of the IPM, boundary flux during testing must be known to integrate instantaneous profiles of water content for obtaining water flow rate. However, it is usually difficult and expensive to measure a boundary flux and if not known, assumptions that may not be easily justified (especially in the field condition) have to be made. In this study, a new method is proposed so that any boundary flux needs not to be measured, controlled or assumed during a test. The new method is evaluated through (i) hypothetical column tests using transient seepage analyses and (ii) five case studies. The new method is capable of determining a SHCF with good accuracy. Normalised root-mean-square deviation (NRMSD) for the old and new methods is less than 5% and 10%, respectively. The accuracy of the new method can be increased substantially (i.e., NRMSD < 5%) when the spacing of sensors installed along a soil column is reduced. Closer sensor spacing reduces error propagation due to numerical differentiation of instantaneous profiles of hydraulic head for determining hydraulic gradient.

KEYWORDS: Hydraulic conductivity, suction, instantaneous profile method, unsaturated soils

Introduction

Soil hydraulic conductivity is one of the vital hydraulic properties that governs transient seepage and affects the magnitude and distribution of pore-water pressure (PWP) and suction in unsaturated soils. This parameter is thus crucial for determining the stability of geotechnical infrastructure such as compacted embankments and slopes subjected to environmental loading such as rainfall (e.g., Ng and Shi, 1998; Leung and Ng, 2013). Soil hydraulic conductivity is also important for agriculturists to determine availability of soil moisture for assessing crop yields (e.g., Wetzel and Chang, 1987; Zhang et al., 2004) as well as ecologists to figure maintenance of ecosystems such as wetlands (e.g., Eldridge and Freudenberger, 2005; Colloff et al., 2010). Unlike saturated soil, hydraulic conductivity of an unsaturated soil is a variable that is not only a function of void ratio but also suction or water content (Romero et al., 1999; Gallage et al. 2013) and net stress (Hung et al., 1998; Ng and Leung, 2012). At a given net stress, the relationship between soil hydraulic conductivity and soil suction or water content is referred to as soil hydraulic conductivity function (SHCF).

Instantaneous profile method (IPM; Watson, 1966; Benson and Gribb, 1997) is a usual transient-state testing method, which is able to directly measure a SHCF that is close to that obtained by a steady-state method but in relatively shorter test duration (Klute, 1972; Fredlund and Rahardjo, 1993; Raoof et al., 2011). The IPM is based on mass continuity to evaluate the instantaneous profiles of water flow rate and hydraulic head under an assumed one-dimensional (1D) water flow condition. According to Darcy's law, SHCF at any depth and elapsed time can then be determined by dividing water flow rate by hydraulic gradient. Based on the original formulation of the IPM adopted by several studies (e.g., Watson, 1966; Hillel et al., 1972), the method works only when any top or bottom boundary condition(s) of a water flow system is measured or assumed during testing because the calculation of water flow rate requires integration. Although top or/and bottom boundary conditions of an 1D soil column under laboratory condition can be controlled and measured relatively conveniently (Watson, 1966; Meerdink et al., 1996; McCartney, 2007; Cui et al., 2008; Ng and Leung, 2012), it may not be the case in the field, where the measurements of boundary flow conditions (if not impossible) are difficult and costly. Assumptions that may not be easily justified had to be made inevitably when using the IPM.

For internal drainage testing method in the field (Rogers and Klute, 1971; Hillel et al., 1972; Basile et al., 2003), it is common to cover the soil surface so that zero boundary flux condition can be assumed at the soil surface. Due to the slow process of internal drainage, another common test method is to not cover the soil surface and to allow for surface evaporation (Plagge et al., 1990; Wendroth et al., 1993; Tse, 2008; Krisdani et al., 2009; Ng et al., 2011; Ng and Leung, 2012). In order to avoid any measurements of actual evaporation rate which is known to be complicated due to the highly-nonlinear soil-atmosphere interaction at the soil surface (Wilson et al., 1997), zero flux is commonly assumed at the bottom boundary (Wendroth et al., 1993; Krisdani et al., 2009; Ng et al., 2011). Such assumption can be unrealistic, and not able to be justified, as no-flux boundary is not well-defined in the field. This could lead to problematic determination of SHCF using the IPM.

In this study, a new alternative interpretation method of the IPM is proposed to determine SHCF without any need to measure, control or assume any boundary flux during testing. The effectiveness of the new method is evaluated through numerical simulations and selected case studies. SHCF obtained by the original method proposed by Watson (1966) and the newly-proposed method is also compared.

Formulation of instantaneous profile method

Fig. 1 shows arbitrary profiles of volumetric water content (VWC; θ) and hydraulic head (H) at elapsed times $t = t_1$ and t_2 along an 1D soil column. The values of VWC and PWP can be measured by, for instances, moisture sensors and suction probes, respectively, at z_A (Row A), z_B (Row B), z_C (Row C) and z_D (Row D). Considering 1D continuity equation in vertical (i.e., z) direction,

$$\frac{\partial v}{\partial z} = -\frac{\partial \theta}{\partial t} \quad (1)$$

where v is water flow rate; θ is VWC; and t is elapsed time. Both v and θ are a function of space (z) and time (t). By Darcy's law, v can be expressed as:

$$v = -k \cdot i = -k \cdot \frac{\partial H}{\partial z} \tag{2}$$

where i is hydraulic gradient; H is hydraulic head, which is a summation of elevation head (z) and pressure head (h); and k is hydraulic conductivity, which is a function of VWC ($k(\theta)$) or pressure head ($k(h)$). By determining v through Eqn (1) and with a known i, k can be obtained by Eqn (2).

Old method

The original approach proposed by Watson (1966) is to determine instantaneous profiles of water flow rate (i.e., v) by integrating Eqn (1) with respect to depth from one end of the soil column, z_E, to the depth under consideration (i.e., $z_{ave} = (z_A + z_B)/2$; Fig. 1). It is an assumption to extrapolate all measured VWC profiles, typically linearly, to the soil surface and the bottom of the soil column (see Fig. 1), if VWC at these two boundaries are not known/measured. Hence, by integration,

$$\int_{z_E}^{z_{ave}} \frac{\partial v}{\partial z} \cdot dz = -\frac{\partial}{\partial t} \int_{z_E}^{z_{ave}} \partial\theta \cdot dz \tag{3}$$

Eqn (3) may be approximated to the following equation:

$$v_{z_{ave},t_{ave}} - v_{z_E,t_{ave}} = -\frac{\Delta V_1}{\Delta t} \tag{4}$$

where $v_{zave,tave}$ is water flow rate at the depth z_{ave} evaluated at the average elapsed time, t_{ave} (=(t_1 + t_2)/2); $v_{zE,tave}$ is the boundary water flow rate evaluated at t_{ave} at the bottom of the soil column (i.e., z_E), either measured or assumed depending on the test conditions; and $\Delta V_1 = V_a + V_b$ (refer to Fig. 1), which can be evaluated by trapezoidal rule. The physical meaning of Eqn (4) is to determine the change of total water volume from depths z_E to z_{ave} between time interval, Δt (= $t_2 - t_1$) in the soil column for a given boundary flux condition. For some testing conditions where only water flow rate at the top boundary of the soil column is known, similar calculation method can be adopted. In this case, the integration in Eqn (3) should be performed from the column surface to the depth of interest.

On the other hand, the hydraulic gradient at depth z_{ave} at t_{ave} ($i_{zave,tave}$) can be evaluated by the central difference method:

$$i_{z_{ave},t_{ave}} = \frac{1}{2}\left[\left(\frac{H_{A,t_1}-H_{B,t_1}}{z_{A,t_1}-z_{B,t_1}}\right)+\left(\frac{H_{A,t_2}-H_{B,t_2}}{z_{A,t_2}-z_{B,t_2}}\right)\right] \tag{5}$$

Hydraulic conductivity at depth z_{ave} and time t_{ave} ($k_{zave,tave}$) can be calculated by dividing the $v_{zave,tave}$ from Eqn (4) by the $i_{zave,tave}$ from Eqn (5), through Darcy's law (Eqn (2)). Hence, a SHCF can then be obtained by relating the calculated $k_{zave,tave}$ with the corresponding suction ($\psi = -h\gamma_w$, where γ_w is the unit weight of water) or θ.

Newly-proposed method

The new method of interpretation of the IPM is to determine $k_{z,t}$ without the need to measure or assume any boundary flux of the soil column in a test. The first step of the proposed method is to integrate the 1D continuity mass equation with respect to depth on both side of Eqn (1) from z_A and z_B, if z_{ave} is the depth of interest:

$$\int_{z_A}^{z_B}\frac{\partial v}{\partial z}\cdot dz = -\frac{\partial}{\partial t}\int_{z_A}^{z_B}\partial\theta\cdot dz \tag{6}$$

Eqn (6) may be approximated to the following equation:

$$v_{z_B,t_{ave}} - v_{z_A,t_{ave}} = -\frac{\Delta V_2}{\Delta t} \tag{7}$$

where $\Delta V_2 = V_b + V_c$ (see Fig. 1). Then, by Darcy's law (Eqn (2)), Eqn (7) can be expressed as:

$$k_{z_B,t_{ave}}\cdot i_{z_B,t_{ave}} - k_{z_A,t_{ave}}\cdot i_{z_A,t_{ave}} = \frac{\Delta V_2}{\Delta t} \tag{8}$$

If the spacing of instruments in the soil column is close enough, it is reasonable to assume that $k_{zA,tave} = k_{zB,tave} = k_{zave,tave}$. Hence, $k_{zave,tave}$ can be calculated using the following equation:

$$k_{z_{ave},t_{ave}} = \frac{\Delta V_2/\Delta t}{i_{z_B,t_{ave}}-i_{z_A,t_{ave}}} \tag{9}$$

It should, however, be noted that the newly-proposed method is not applicable for any flow condition that gives zero or uniform hydraulic gradient (i.e., $i_{zA,tave} = i_{zB,tave}$). These cases would result in numerical singularity in Eqn (9). Moreover, the $i_{zA,tave}$ and $i_{zB,tave}$ must have the same sign to satisfy the mass continuity. As k is always positive, $i_{zA,tave}$ must be larger than $i_{zB,tave}$ when the soil column is subjected to drying process (i.e., negative ΔV_2), whereas $i_{zA,tave}$ must be smaller than $i_{zB,tave}$ when wetting process is considered (i.e., positive ΔV_2).

Methods of evaluation

Two phases of evaluation are conducted to examine the effectiveness of the newly-proposed interpretation method, compared to the old approach. The first phase is to carry out hypothetical soil column tests through numerical simulation, similar to the approach adopted by Chen et al. (2010). By applying different known boundary flux conditions and with known soil hydraulic properties including SHCF (denoted as $SHCF_{input}$), instantaneous profiles of VWC and hydraulic head can be determined. Then, using Eqns (3) – (9), the two computed profiles can be used to determine SHCF by the newly-proposed and old methods (denoted as $SHCF_{new}$ and $SHCF_{old}$, respectively). In order to evaluate the accuracy of $SHCF_{new}$ and $SHCF_{old}$ as compared to the known $SHCF_{input}$, normalized root-mean-square deviation (NRMSD) is used. It is a statistical means of precision measure of any deviations between values estimated by two different methods. NRMSD can be calculated by the following equation:

$$NRMSD_{m,input} = \frac{\left\{\frac{1}{N}\sum_{\psi=0.1}^{80}\left[log(k_m(\psi)) - log\left(k_{input}(\psi)\right)\right]\right\}^{1/2}}{log(max\{k_m(\psi), k_{input}(\psi)\}) - log(min\{k_m(\psi), k_{input}(\psi)\})} x100\% \qquad (10)$$

where N is the number of observations; ψ is matric suction; $k_m(\psi)$ is either $k_{old}(\psi)$ or $k_{new}(\psi)$, which refers to $SHCF_{old}$ and $SHCF_{new}$, respectively. NRMSD is evaluated for suctions ranged from 0.1 to 80 kPa. A lower NRMSD indicates a better agreement between $SHCF_{input}$ and $SHCF_{new}$ (or $SHCF_{old}$). In the ideal condition of perfect agreement, NRMSD is equal to zero.

The second phase of evaluation is through case studies reported in the literature. Test data from three laboratory (Watson, 1966; McCartney 2007; Ng and Leung, 2012) and two field (Hillel et al., 1976; Ng et al., 2011) studies are selected for re-interpretation using the new method. $SHCF_{old}$ in all cases is known. The evaluation is conducted by investigating the degree of agreement between $SHCF_{new}$ and $SHCF_{old}$ using NRMSD. The equation of NRMSD in this case is slightly different from the previous case and is expressed as:

$$NRMSD_{new,old} = \frac{\left\{\frac{1}{N}\sum_{\psi=0.1}^{80}\left[log(k_{new}(\psi)) - log\left(k_{old}(\psi)\right)\right]\right\}^{1/2}}{log(max\{k_{new}(\psi), k_{old}(\psi)\}) - log(min\{k_{new}(\psi), k_{old}(\psi)\})} x100\% \qquad (11)$$

Eqn (11) is to determine the degree of agreement between $SHCF_{new}$ and $SHCF_{old}$, while Eqn (10) is to compare $SHCF_{input}$ with either $SHCF_{new}$ or $SHCF_{old}$.

Datasets for evaluation

Phase 1: numerical simulation

A commercial finite element package, SEEP/W (Geo-Slope Int., 2009), is used to simulate two hypothetical SHCF tests, namely internal drainage test and evaporation test, which are the two most common test methods adopted by various existing field and laboratory studies. For both types of test, a 1-m high 1D sandy soil column is simulated. The soil water retention curve (SWRC) and SHCF of the sandy material are shown in Fig. 2. When simulating the internal drainage test, the sand column is initially fully saturated (i.e., uniform zero suction) and is allowed to drain. In terms of modelling, no-flux boundary is specified on the top of the column surface to simulate the covering of soil surface (i.e., no evaporation took place), while unit-gradient flux boundary condition is applied at the column base to simulate free drainage condition. This flux boundary means that the nodal discharge flux at the column base was fixed to be equal to $-k$ (ψ), according to Darcy’s law. The test duration is set to be 60 days, which is long enough to develop a range of suction between 0 – 80 kPa for evaluation. Figures 3(a) and (b) show the 7-day isochrones of the computed instantaneous profiles of VWC and PWP during the drainage process. Although continuous profiles can be obtained from numerical simulation, in field or laboratory measurements these profiles are normally obtained by sensors installed at discrete depths. The computed profiles shown in Fig. 3 are therefore discretised, considering that four rows of VWC and PWP sensors are installed at uniform depths of 0.12, 0.38, 0.62 and 0.88 m depths along the 1 m-high column. It can be seen in Fig. 3(a) that the sand column, which has an initial saturated VWC of 0.4, exhibits uniform reduction of VWC, as water drains at the column base. This causes uniform decreases in PWP (or increases in suction), reaching about -70 kPa at the end of the hypothetical test (Fig. 3(b)). Eqns (3) – (9) are then applied to these discretised profiles for determining $SHCF_{new}$ and $SHCF_{old}$. In order to explore the effects of sensor spacing on the newly-proposed method, the calculation of $SHCF_{new}$ is repeated using more densely-discretised VWC and PWP profiles, considering 10 pairs of sensors uniformly distributed at 0.06, 0.16, 0.26, 0.36, 0.46, 0.56, 0.66, 0.76, 0.86 and 0.96 m depths (100 mm spacing).

For the evaporation test, the sand column is also fully saturated initially. In this case, the top surface of the column is subjected to 30-day evaporation process, while free drainage (i.e., unit-gradient flux boundary) is allowed at the column base. A simplified modelling approach of evaporation is adopted. Instead of modelling the nonlinear soil-atmosphere interaction (Wilson et al., 1997), the effect of evaporation is simulated by applying a constant negative pressure head of -12 m at the column surface. It should be noted that the objective of this numerical simulation is not to investigate the soil-atmosphere interaction, but to obtain different shapes of instantaneous profiles of VWC and hydraulic head due to different applied boundary conditions for evaluating the new method. Such simplified modelling method is considered to be acceptable. The evaporation rate at the column surface and the drainage rate at the column base are obtained for determining SHCF using the old method (i.e., the term $v_{zE,tave}$ in Eqn (4)). The 5-day isochrones of the computed instantaneous profiles of VWC and PWP during the evaporation are depicted in Figs 3(c) and (d), respectively. It can be seen in Fig. 3(c) that the sand column loses moisture near the top surface via evaporation in a rate faster than that near the base due to drainage. Compared to the 60-day internal drainage test, more soil moisture is lost in the evaporation test. This thus leads to higher suction induced in the soil column at the end of the test, even though the test duration (30 days) is halved.

Phase 2: selected case studies

Three laboratory (Watson, 1966; McCartney, 2007; Ng and Leung, 2012) and two field (Hillel et al., 1972; Ng et al., 2011) studies are selected for evaluation. The tests reported by Watson (1966), McCartney (2007) and Hillel et al. (1972) are internal drainage tests, while those by Ng and Leung (2012) and Ng et al. (2011) are evaporation test. These five case studies are chosen because they document comprehensive test datasets and most of the necessary information that can be used to evaluate both the old and the new methods.

Watson (1966) performed a laboratory column test for sand with 0.57 m in height. The sand column was initially water saturated. Before testing, a small head of water of 1 mm was maintained at the column surface and the base of the column was opened to atmosphere for allowing free bottom drainage. At equilibrium, the drainage test commenced when the small ponded water disappeared through the column surface. During the drainage process, instantaneous profiles of VWC and suction were obtained by destructive methods using replicated sand columns (exact procedures not reported). By relating the VWC and suction measured during the test, the soil water retention curve (SWRC) of the sand is obtained (Fig. 4(a)). The air-entry value (AEV) is ~4 kPa, beyond which a very significant reduction of VWC from 0.36 to 0.09 is observed.

McCartney (2007) measured a SHCF of loosely-compacted clay using a 0.75 m-height column, where a constant water inflow rate of 8 x 10^{-8} m/s was applied at the column surface in the laboratory. Free water drainage was allowed at the column base during testing. After compaction, the clay column was initially unsaturated, with a uniform distribution of VWC of 14.5%. Upon infiltration, instantaneous profiles of VWC and any outflow rate at the column base were monitored. Pore-water pressure (or suction) was not recorded during the drainage process, and it was estimated by mapping the measured VWC to a SWRC, which was separately measured by a pressure plate (Fig. 4(a)). The test was lasted for almost 63 days (i.e., 1500 hours). However, the calculation performed in this note only involves the results that cover the first 25 days (i.e., 600 hours) of the test. This is because on the 25th day, the wetting front has advanced to the column base and a steady-state condition has reached. Therefore, only the transient flow process between days 0 to 25 is relevant for carrying out the IPM calculation (old or new method).

Ng and Leung (2012) developed a stress-controlled soil column apparatus to measure SHCFs of compacted silty clay under three different vertical load levels, 0, 40 and 80 kPa, in laboratory. Each silty clay column was saturated and then loaded to the desired stress level. At equilibrium, the top surface of each column was subjected to soil evaporation for approximately 130, 175 and 260 days, whereas the bottom flux was controlled to be zero by closing a valve installed near each column base. Four pairs of soil moisture sensors and tensiometers were installed at 200, 325, 700 and 825 mm depths. As shown in Fig. 4(a), SWRC of the compacted soil is stress-dependent. As vertical load increases, the silty clay has increased ability to retain moisture, higher AEV and lower desorption rate. In their study, evaporation was not measured. Therefore, the determination of SHCFs through the old method had to be carried out by integrating the measured VWC profiles from the column base, where the boundary flux (i.e., $v_{zE,tave}$) was known (i.e., zero flux for each test).

A field SHCF test was conducted by Hillel et al. (1972) on a flat ground comprising alluvial sandy-loam soil. More than one neutron probes (exact number not reported) and five tensiometers were installed to monitor the responses of VWC and suction, respectively, at average depths of 0.3, 0.6, 0.9, 1.2 and 1.5 m. Due to the ground variability in the field, the soil water retention ability was found to be different at different depths (Fig. 4(b)). The test was started by irrigating the plot until all sensors recorded steady-state values, and then followed by an internal drainage test. During the test, the plot was covered with a plastic sheet to prevent any water flux across the surface. This test method hence created no-flux top boundary condition for the old method to determine SHCF.

Ng et al. (2011) carried out a similar field SHCF tests as Hillel et al (1972) but on a test site consisting of a 1 m-thick colluvium (sandy silt) layer overlying a 3-m thick decomposed silty clay stratum. In the site studied by Ng et al. (2011), a bedrock surface was encountered below 4 m depth. They installed ten tensiometers at 0.36 to 2.99 m depths, but only four time domain reflectometers (TDR) for monitoring VWC due to budget constraints. This study consisted of two wetting-drying cycles. The 1st phase was to pond the site until steady-state readings were recorded, and then followed by an evaporation test in the 2nd phase for 24 days. The plot was dried by both soil evaporation at the surface and internal drainage. The second wetting (3rd phase)-drying (4th phase) cycle was the repetition of the first cycle, but the test duration for the second drying cycle was shortened to 16 days. Based on the test data reported by Ng et al. (2011), only the responses of VWC and PWP within the top 1.2 m depth showed significant change, and thus only SHCF of colluvium is assessed in this note. The SWRC of the colluvial sandy silt shown in Fig. 4(b) indicates that the AEV is ~1 kPa. Ng et al. (2011) reported that the colluvial deposit in the field is rather loose as a result of historical mass wasting process. It is thus not surprising for the loose colluvium to have a low AEV. As suction increases beyond 4 kPa, it appears that a residual VWC of 0.25 has reached.

Table 1 summarises the test conditions of the five selected case studies for evaluation. The measured instantaneous profiles of VWC and PWP of the five case histories are depicted in Figs A1 to A5 in the appendix. The measurements are not discussed in this note, as all details have been reported in the respective studies. The purpose of showing these instantaneous profiles is to help evaluate the effectiveness of the newly-proposed interpretation method.

Results and discussion

Figure 5 compares the $SHCF_{input}$ with both the $SHCF_{old}$ and $SHCF_{new}$ for the hypothetical internal drainage test (Fig. 5(a)) and the evaporation test (Fig. 5(b)). Within the range of suction investigated, the SHCF determined by the old method (i.e., $SHCF_{old}$) almost resembles the "true" SHCF (i.e., $SHCF_{input}$) in both cases ($NRMSD_{old,input}$ lower than 2%; Table 2), when the boundary flux is available for the calculation using Eqn (4). Although the newly-proposed method does not give as good estimation as the old method, the $NRMSD_{new,input}$ is considerably low (i.e., < 9.5%), given the fact that the calculation using the new method did not require any boundary flux. To further interpret the calculation, all $SHCF_{new}$s are fitted with the equation proposed by van

Genuchten (1980). It can be seen that there is a slight shift of the fitted $SHCF_{new}$ from the $SHCF_{input}$. For both hypothetical tests, the fitting parameter, *a*, appears to change negligibly (Table 3). Despite of a noticeable difference of the parameter, *n* (which indicates the reduction rate of hydraulic conductivity for a given suction increase) for the internal drainage test (Fig. 5(a)), the discrepancy reduces significantly when using 10 rows of sensors. The fitting parameters for this modified case are close to those for $SHCF_{input}$. Correspondingly, $NRMSD_{new,input}$ reduces to 3.3% only (Table 2). When using more sensors to obtain hydraulic head profiles, the error of propagation due to numerical differentiation (for determining hydraulic gradient profiles; Eqn (5)) would be reduced (Fluhler et al., 1976; Zhang et al., 2012), hence increasing the accuracy of SHCF estimation.

Comparisons of $SHCF_{old}$ and $SHCF_{new}$ for the five selected case histories are shown in Fig. 6. Good agreements between $SHCF_{old}$ and $SHCF_{new}$ for both the laboratory and field internal drainage tests reported by Watson (1966), Hillel et al. (1972) and McCartney (2007) are sought (Figs 6(a), (b), (c) and 7). The values of $NRMSD_{old,new}$ for these three cases are not more than 12% (Table 2). In these three selected case studies, both the $SHCF_{old}$ and $SHCF_{new}$ are able to be fitted by using the same set of parameters of van Genuchten (1980)'s equation reasonably well (Table 3). Note that the difference of the steady-state and transient-state SHCF of the clay reported by McCartney (2007; Fig. 6(c)) is because PWP in the old IPM calculation is deduced indirectly from the drying SWRC (Fig. 4), rather than from direct measurement. However, it must be pointed out that the aim of this note is to study the effectiveness of the newly-proposed SHCF estimation method. The accuracy of measuring PWP is not relevant to this study. As long as the same set of column test data is used, the comparison of SHCF estimated by the old and newly-proposed method would be valid.

Despite of some scattering of SHCFs for the case by Ng and Leung (2012; Fig. 6(d)), the $SHCF_{new}$ for each stress level exhibits a similar trend to the corresponding $SHCF_{old}$, resulting in an acceptable range of $NRMSD_{old,new}$ from 12% to 17% (Table 2). For each stress level, the same set of fitting parameters appears to be capable of capturing both the $SHCF_{old}$ and $SHCF_{new}$ (Table 3).

For the case Ng et al. (2011), the new method gives a $SHCF_{new}$ that has a greater decreasing rate of hydraulic conductivity than the $SHCF_{old}$, during both the 1st and 2nd drying phase of the tests (Fig. 6(e)). As compared to the old method, the hydraulic conductivity determined by the new method can be under-estimated by up to 60% (see Fig. 7). Such discrepancy hence led to relatively high $NRMSD_{old,new}$ of 20% to 24% (Table 2). It must be pointed out that in this particular field study, estimation of water flow rate in the colluvium is less accurate, regardless of the use of the old or the new method, as only one TDR was installed in the colluvium at 0.84 m depth (Table 1 and Fig. A5). In order to determine ΔV_1 using Eqn (4) and ΔV_2 using Eqn (9), one assumption that must be made is to extrapolate all instantaneous profiles of VWC from the depth of 0.84 to 0.77 m, where the closest tensiometer was installed in the colluvium near the TDR. As this specific limitation affects the accuracy of both the old and the new method, the values of $NRMSD_{old,new}$, obtained for this particular case should not be relied on, when attempting to evaluate the performance of the newly-proposed method.

Summary and conclusions

This note proposes a new interpretation method of the instantaneous profile method for determining a hydraulic conductivity function of an unsaturated soil. The new method has a major advantage over the old method by removing any need to measure and assume either the top or bottom boundary flux, which is usually expensive and difficult to obtain in a test. The new method is evaluated through (i) hypothetical column tests using finite element transient seepage analyses and (ii) five selected case studies. These selected case studies document comprehensive test datasets that are sufficient for re-interpretation. In this note, internal drainage test (i.e., no flux or ponding at top boundary and free bottom drainage) and evaporation test (i.e., evaporation at top boundary and free drainage or no-flux bottom boundary) are used to examine the accuracy of the estimation of soil hydraulic conductivity function (SHCF) using the old and new methods.

The new method is revealed to have good accuracy in determining a SHCF for both types of tests without any use of measured/assumed boundary flux. This is indicated by the low normalised root-mean-square deviation (NRMSD; lower than 10%). The accuracy is further increased (i.e., NRMSD drops below 5%) when the spacing of sensors installed in a soil column is decreased. Closer sensor spacing reduces error propagation due to differentiation of hydraulic head profiles when determining hydraulic gradient. No major difference is found between the SHCFs determined by the old and new methods (NRMSD not more than 17%), expect for one field evaporation test where there is a lack of water content measurement for the calculation of water flow rate (which gives a NRMSD up to 25%). The new method works especially well for flow conditions that do not result in zero or constant hydraulic gradient, where numerical singularity would occur during SHCF calculation.

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References

Basile, A., Ciollaro, G., and Coppola, A. 2003. Hysteresis in soil water characteristics as a key to interpreting comparisons of laboratory and field measured hydraulic properties. Water Resources Research, **39**(12): 1355 – 1366.

Benson, C.H., and Gribb, M. M. 1997. Measuring unsaturated hydraulic conductivity in the laboratory and field. In: Unsaturated Soil Engineering Practice, New York: ASCE, 0-7844-0259-0, 1997, 344 pp., Geotechnical Special Publication No. 68, 113 – 168.

Chen, R., Li, Z.H., and Li, J.H. 2010. Discussion of "Wetting front advancing column test for measuring unsaturated hydraulic conductivity". Canadian Geotechnical Journal, **47**(10): 1159 – 1161.

Colloff, M.J., Pullen, K.R., and Cunningham, S.A. 2010. Restoration of an ecosystem function to revegetation communities: the role of invertebrate macropores in enhancing soil water infiltration. Restoration Ecology, **18**(s1): 65–72.

Cui, Y.J., Tang, A-M., Loiseau, C., and Delage, P. 2008. Determining the unsaturated hydraulic conductivity of a compacted sand-bentonite mixture under constant volume and free-swell conditions. Physics and Chemistry of the Earth, **33**: S462 – S471.

Eldridge, D.J., and Freudenberger, D. 2005. Ecosystem wicks: woodland trees enhance water infiltration in a fragmented agricultural landscape in eastern Australia. Austral Ecology, **30**(3): 336–347.

Fluhler, H., Ardakani, M.S., and Stolzy, L.H. 1976. Error propagation in determining hydraulic conductivities from successive water content and pressure head profiles. Soil Science Society of America Journal, **40**: 830 – 836.

Fredlund, D.G., and Rahardjo, H. 1993. Soil mechanics for unsaturated soils. Wiley Inter Science, New York.

Gallage, C.P., Kodikara, J.K., and Uchimura, T. 2013. Laboratory measurement of hydraulic conductivity functions of two unsaturated sandy soils during drying and wetting processes. Soils and Foundations, **53**(3): 417 – 430.

Geo-Slope International Ltd. 2009. Seepage Modeling with SEEP/W, An Engineering Methodology, 4th Edition

Hillel, D., Krentos, V.D., and Stylianou, Y. 1972. Procedure and test of an internal drainage method for measuring soil hydraulic characteristic in-situ. Soil Science, **114**(5): 395 – 400.

Hung, S., Fredlund, D.G., and Babour, S. L. 1998. Measurement of the coefficient of permeability for a deformable unsaturated soil using a triaxial permeameter. Canadian Geotechnical Journal, **35**(3): 426 – 432.

Klute, A. 1972. The determination of the hydraulic conductivity and diffusivity of unsaturated soils. Soil Science, **113**: 264-276.

Krisdani, H., Rahardjo, H., and Leong, E. C. 2009. Use of instantaneous profile and statistical methods to determine permeability functions of unsaturated soils. Canadian Geotechnical Journal, **46**: 869 – 874.

Leung, A.K., and Ng, C.W.W. 2013. Analysis of groundwater flow and plant evapotranspiration in a vegetated soil slope. Canadian Geotechnical Journal, **50**(12): 1204 – 1218.

McCartney, J.S. 2007. *Determination of the hydraulic characteristics of unsaturated soils using a centrifuge permeameter*. PhD Dissertation. The University of Texas at Austin.

Meerdink, J.S., Benson, C.H., and Khire, M.V. 1996. Unsaturated hydraulic conductivity of two compacted barrier soils. Journal of Geotechnical Engineering, ASCE, **122**(7): 565 – 576.

Ng, C.W.W., and Leung, A.K. 2012. Measurements of drying and wetting permeability functions using a new stress-controllable soil column. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, **138**(1): 58 – 65.

Ng, C.W.W., and Shi, Q. 1998. A numerical investigation of the stability of unsaturated soil slopes subjected to transient seepage. Computers and Geotechnics, **22**(1): 1 – 28.

Ng, C.W.W., Wong, H.N., Tse, Y.M., Pappin, J.W., Sun, H.W., Millis, S.W., and Leung, A.K. 2011. Field study of stress-dependent soil-water characteristic curves and hydraulic conductivity in a saprolitic slope. Geotechnique, **61**(6): 511 – 521.

Plagge. R., Renger, M., and Roth, C.H. 1990. A new laboratory method to quickly determine the unsaturated hydraulic conductivity of undisturbed soil cores within a wide range of texture. Journal of Plant Nutrient and Soil Science, **153**(1): 39 – 45.

Raoof, M., Nazemi, A.M., Sadraddini, A.A., and Marofi, S. 2011. Measuring and estimating saturated and unsaturated hydraulic conductivity in steady and transient states on sloping lands. World Applied Sciences Journal, **13**(4): 747 – 755.

Rogers, J.S., and Klute, A. 1971. The hydraulic conductivity-water content relationship during nonsteady flow through a sand column. Soil Science Society of America Journal, **35**(5): 695–700.

Romero, E., Gens, A., and Lloret, A. 1999. Water permeability, water retention and microstructure of unsaturated compacted boom clay. Engineering Geology, **54**(1–2): 117 – 127.

Tse. Y.M. 2008. Laboratory and field studies of drying-wetting effects on shear strength, SDSWCC and permeability of an unsaturated saprolite. MPhil Thesis, the Hong Kong University of Science and Technology.

van Genuchten, M.Th. 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Science Society of America Journal, **44**: 892 – 898.

Watson, K.K. 1966. An instantaneous profile method for determining the hydraulic conductivity of unsaturated porous materials. Water Resources Research, **2**(4): 709 – 715.

Wendroth, O., Ehlers, W., Hopmans, W., Kage, H., Halbertsma, J., and Wosten, J.H.M. 1993. Reevaluation of the evaporation method for determining hydraulic functions in unsaturated soils. Soil Science Society of America Journal, **57**: 1436 – 1443.

Wetzel, P.J., and Chang, J.T. 1987. Concerning the relationship between evapotranspiration and soil moisture. Journal of Applied. Meteorology and Climatology, **26**(1): 18–27.

Wilson, G.W., Fredlund, D.G., and Barbour, S.L. 1997. The effect of soil suction on evaporation fluxes from soil surfaces. Canadian Geotechnical Journal, **34**(1): 145 – 155.

Zhang, Y., Kendy, E., Qiang, Y., Liu, C., Shen, Y., and Sun, H. 2004. Effect of soil water deficit on evapotranspiration, crop yield, and water use efficiency in the North China Plain. Agricultural Water Management, **64**(2): 107–122.

Zhang, Y.L., Chen, R., Li, J.H., and Xu, J. 2011. Evaluating the accuracy of unsaturated hydraulic conductivity determined by instantaneous profile method. Proceedings of the 5th Asia-Pacific Conference on Unsaturated Soils, Kasetsart University, Pattaya, Thailand, Jotisankasa, Sawangsuriya, Soralump and Mairaing, eds., 403 – 407.

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Table 1. A summary of test conditions of the five selected case studies

Case	Watson (1966)	McCartney (2007)	Ng and Leung (2012)	Hillel et al. (1972)	Ng et al. (2011)
Field/Lab	Lab	Lab	Lab	Field	Field
Soil type	Botany sand	Clay	Compacted silty clay	Alluvial sandy-loam	Colluvium
Test method	Internal drainage	Internal drainage	Evaporation	Internal drainage	Evaporation
Thickness of soil tested	0.57 m	0.75 m	1 m	1.5 m	1 m
Top boundary condition	Covered (no-flux)	Ponding (applied constant influx)	Exposed (evaporation *not* measured)	Covered (no-flux)	Exposed (evaporation *not* measured)
Bottom boundary condition	Freely drained (drainage rate *measured*)	Freely drained (drainage rate *measured*)	Closed (no-flux)	Freely drained (drainage rate *not* known)	
Test duration	20 min	25 d	0 kPa: 130 d 40 kPa: 175 d 80 kPa: 260 d	25 d	1st drying: 24 d 2nd drying: 16 d
Instrument depths	Not reported, but suction and VWC profiles for the entire 0.57 m-high column are presented	**VWC:** 0.05, 0.25, 0.35, 0.5, 0.65 and 0.7 m **Suction** estimated from SWRC	**Suction and VWC:** 0.2, 0.325, 0.7 and 0.825 m	**Suction and VWC**: 0.3, 0.6, 0.9, 1.2 and 1.5 m	**Suction**: 0.36, 0.77, 0.95, 1.17, 1.54, 1.85, 2.13, 2.43, 2.6 and 2.9 m **VWC**: 0.84, 1.85, 2.5, 3.59 m

Table 2. NRMSD of SHCF determined by the old and new methods

Case	NRMSD
Hypothetical column tests	
Internal drainage test (sensor spacing of 100 mm)	
$NRMSD_{old,input}$	0.11%
$NRMSD_{new,input}$	3.30%
Internal drainage test (sensor spacing of 250 mm)	
$NRMSD_{old,input}$	0.54%
$NRMSD_{new,input}$	9.49%
Evaporation test (sensor spacing of 250 mm)	
$NRMSD_{old,input}$	1.81%
$NRMSD_{new,input}$	5.74%
Case studies ($NRMSD_{new,old}$)	
Watson (1966) – Lab internal drainage test	11.94%
Hillel et al. (1972) – Field internal drainage test	10.28%
McCartney (2007) – Lab internal drainage test	9.18%
Ng and Leung (2012) – Lab evaporation test	
Vertical load of 0 kPa	12.42%
Vertical load of 40 kPa	12.04%
Vertical load of 80 kPa	16.52%
Ng et al. (2011) – Field evaporation test	
1st drying cycle	19.15%
2nd drying cycle	23.57%

Table 3. Summary of the fitting coefficients of van Genuchten (1980)'s equation for SHCF

Case	Fitting parameters				
	a **(kPa^{-1})**	***n***	***m*** **= 1 – 1/n***	***l****	**k_s (m/s)**
Hypothetical column tests					
$SHCF_{input}$	0.065	1.64	0.390	0.5	5 x 10^{-6}
Internal drainage test (4 sensors)	0.053	1.90	0.473		
Internal drainage test (10 sensors)	0.060	1.64	0.390		
Evaporation test	0.071				
Case studies					
Watson (1966)	0.230	22.10	0.955	0.5	1.84 x 10^{-6}
Hillel et al. (1972) – 0.6 m	0.070	4.10	0.756		2.24 x 10^{-6}
Hillel et al. (1972) – 0.9 m	0.087	3.08	0.675		
Hillel et al. (1972) – 1.2 m	0.085	2.50	0.600		
McCartney (2007) – Steady-state	0.025	3.51	0.715		8 x 10^{-7}
McCartney (2007) – Transient-state	0.064	1.92	0.479		
Ng et al. (2012) – 0 kPa	0.020	1.40	0.286		5 x 10^{-8}
Ng et al. (2012) – 40 kPa	0.025	1.45	0.310		2 x 10^{-8}
Ng et al. (2012) – 80 kPa	0.010	1.52	0.342		2 x 10^{-9}
Ng et al. (2011) – 1st drying	0.350	2.61	0.617		3.5 x 10^{-5}
Ng et al. (2011) – 2nd drying	0.42	3.23	0.690		3.5 x 10^{-5}

*According to van Genuchten (1980)

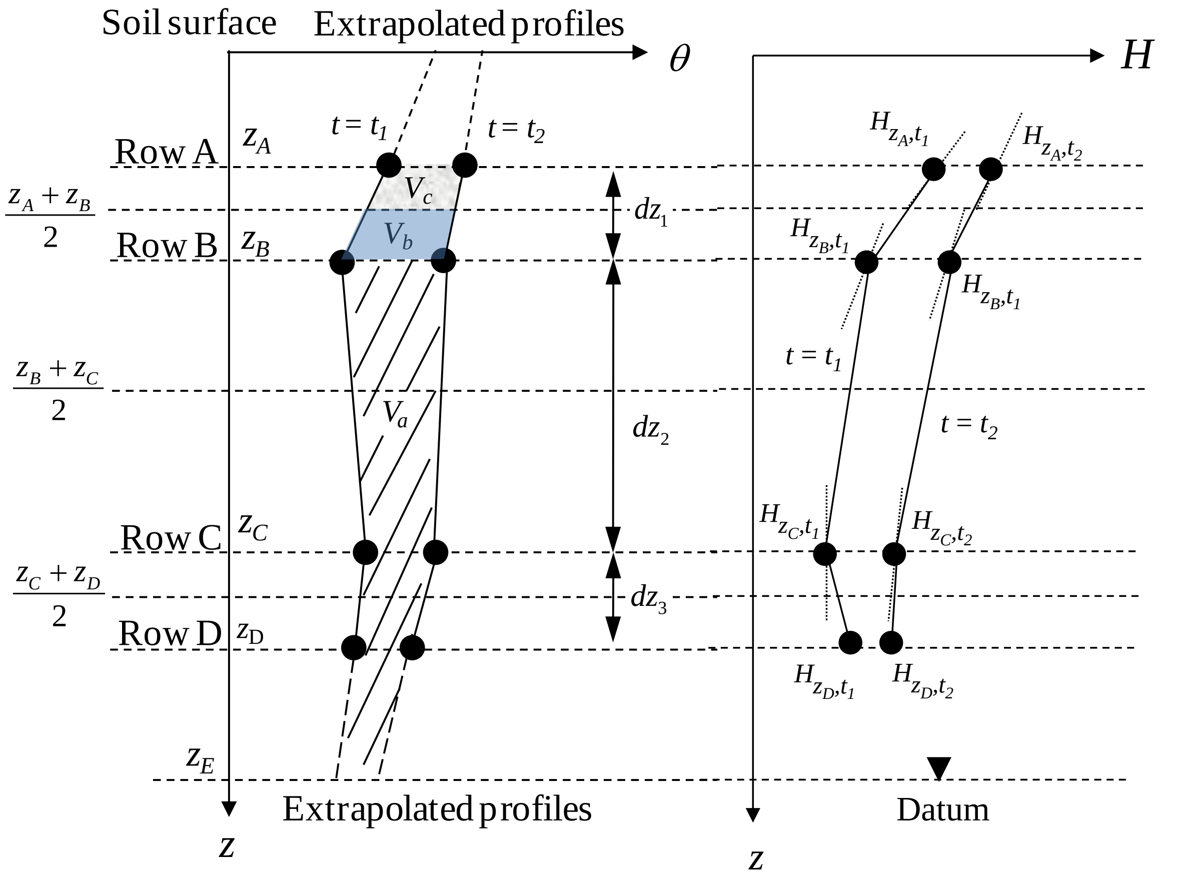


Figure 1. Arbitrary instantaneous profiles of VWC (θ) and hydraulic head (H) at elapsed times, $t = t_1$ and t_2, along an one-dimensional soil column

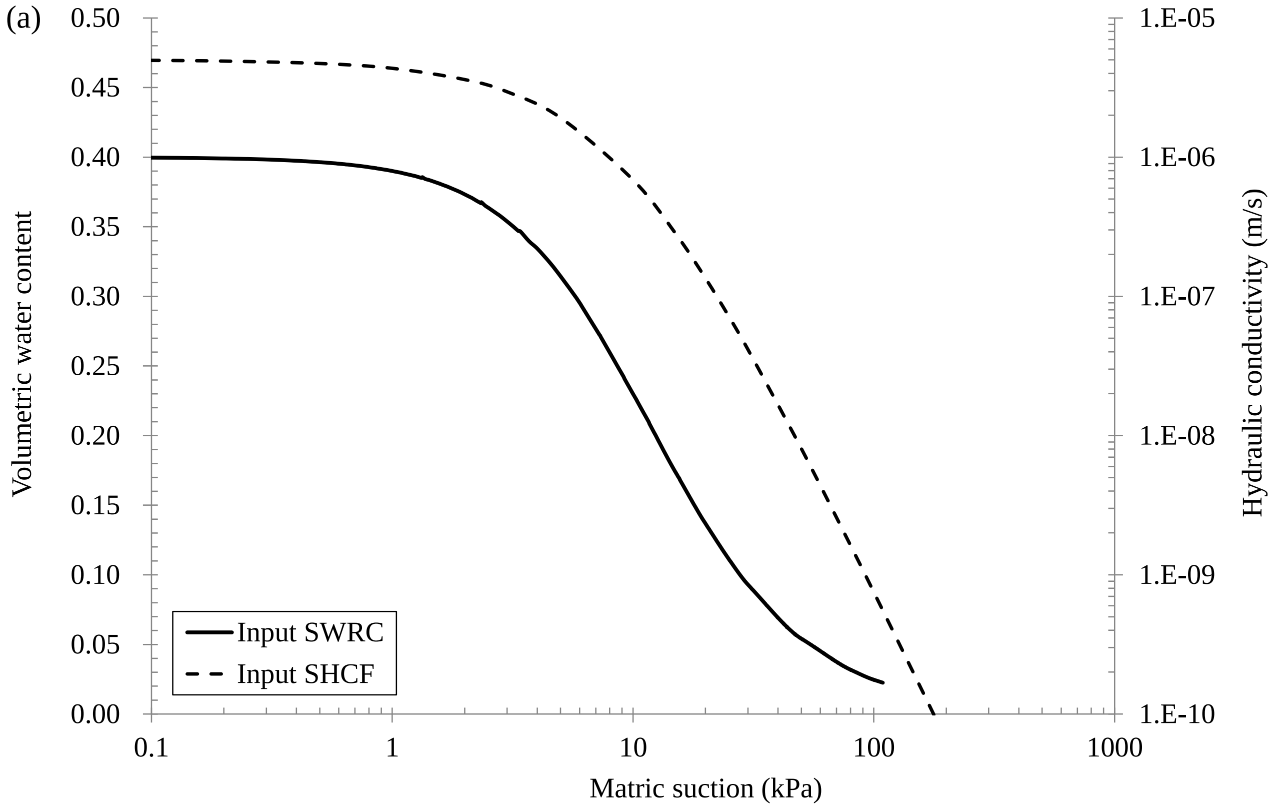
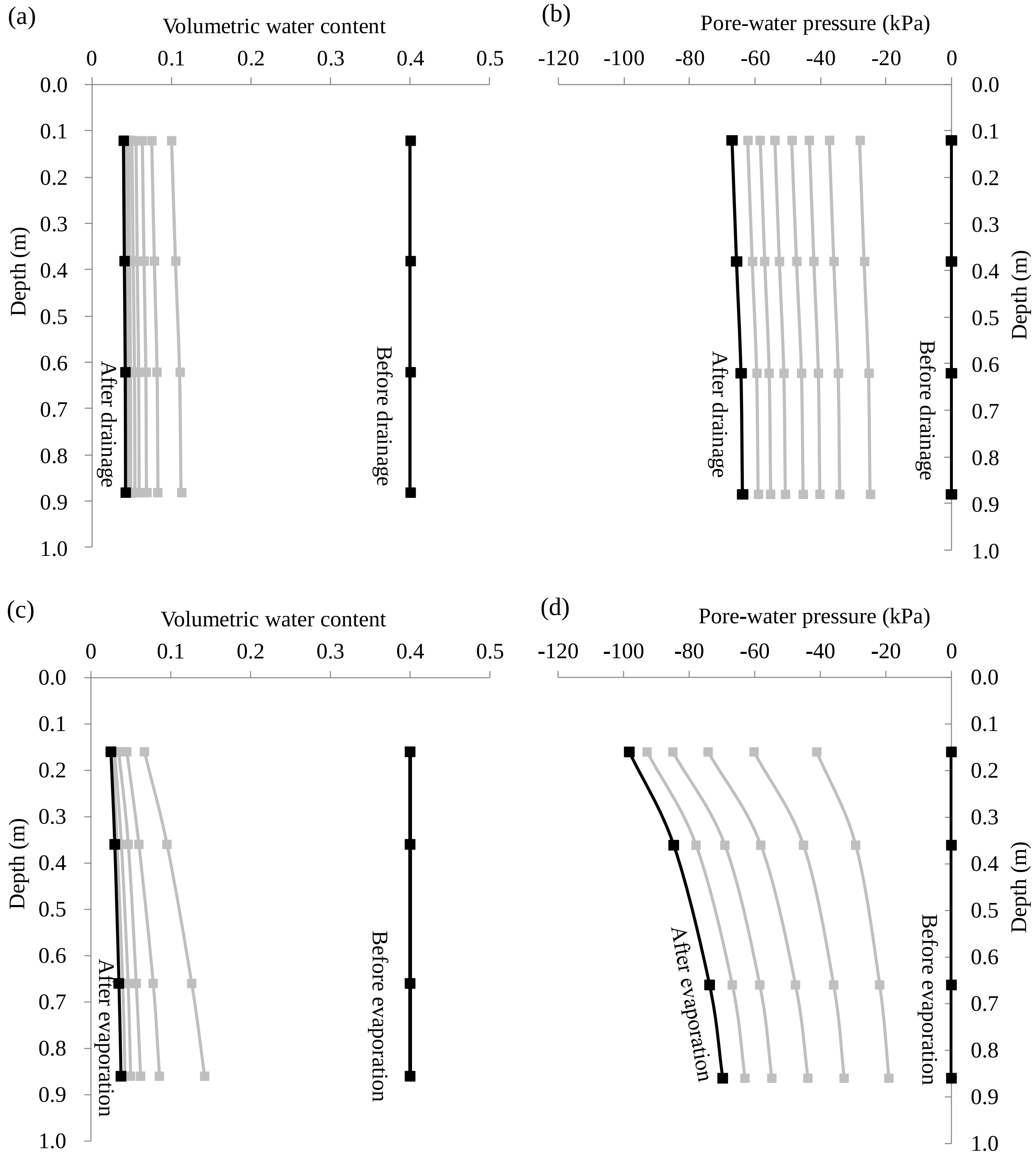


Figure 2. Input soil water retention curve (SWRC) and soil hydraulic conductivity function (SHCF) of sand for the hypothetical column tests



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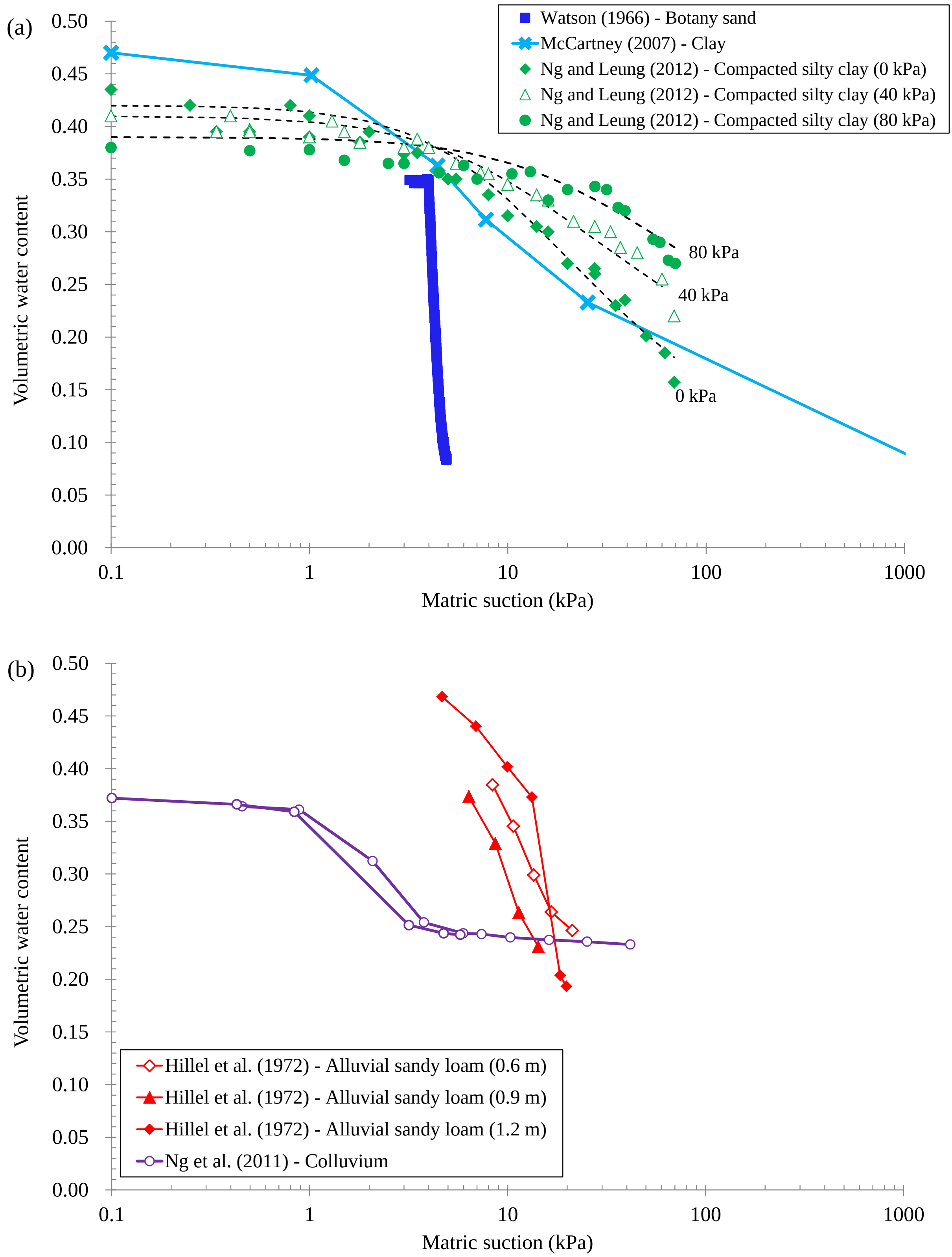

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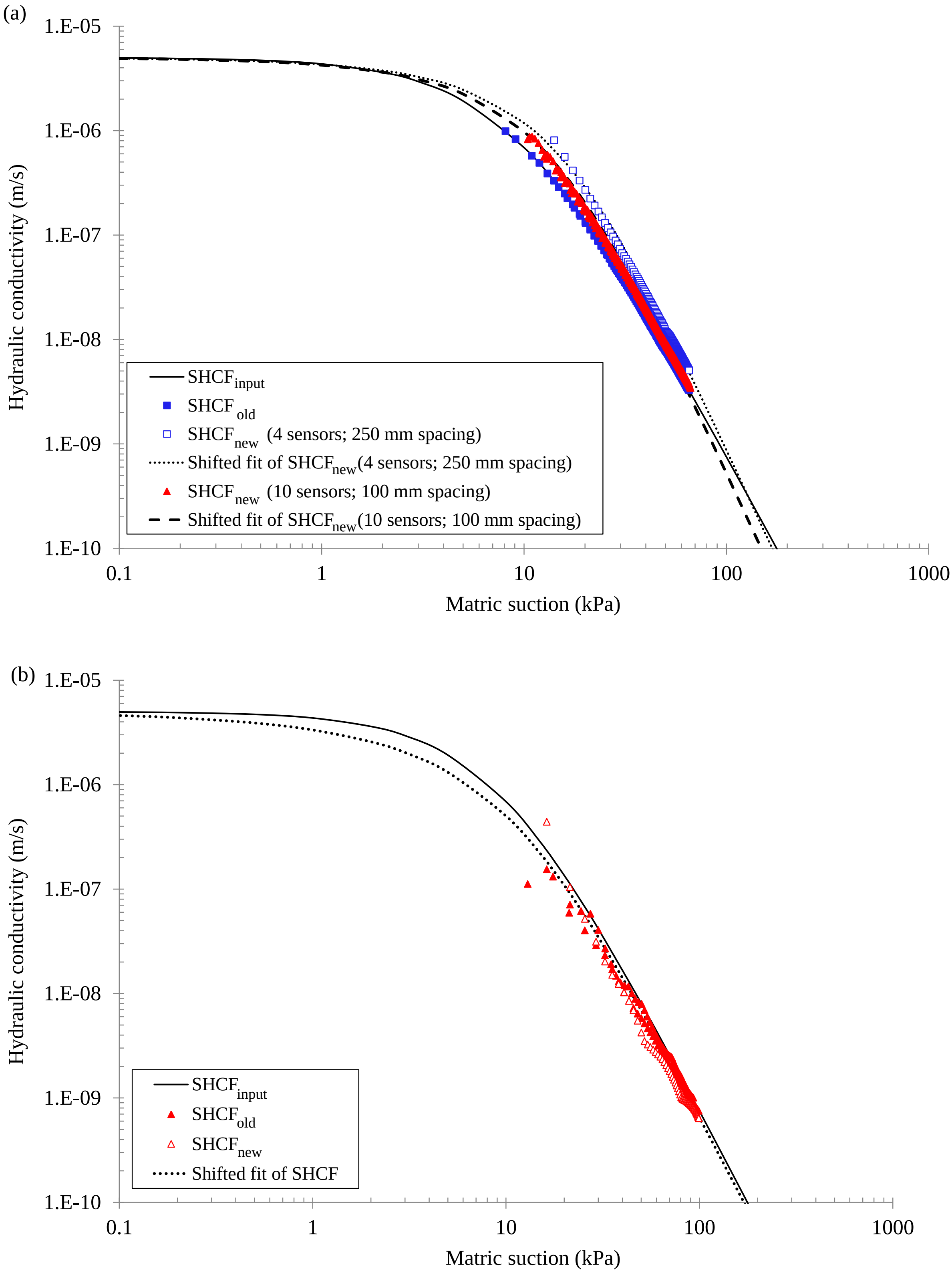


Figure 5. Comparisons of $SHCF_{input}$, $SHCF_{old}$ and $SHCF_{new}$ obtained from the hypothetical (a) internal drainage test and (b) evaporation test

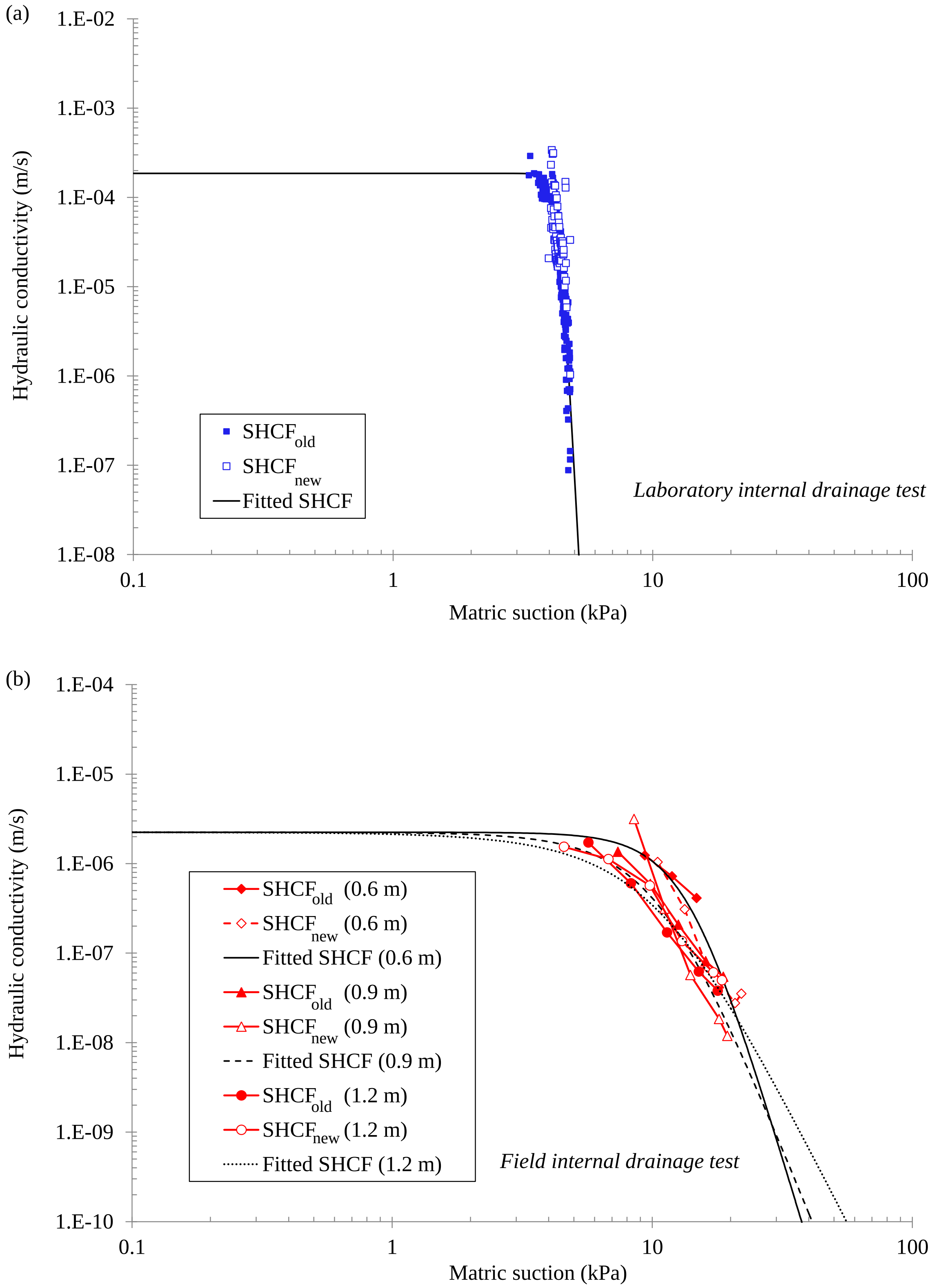
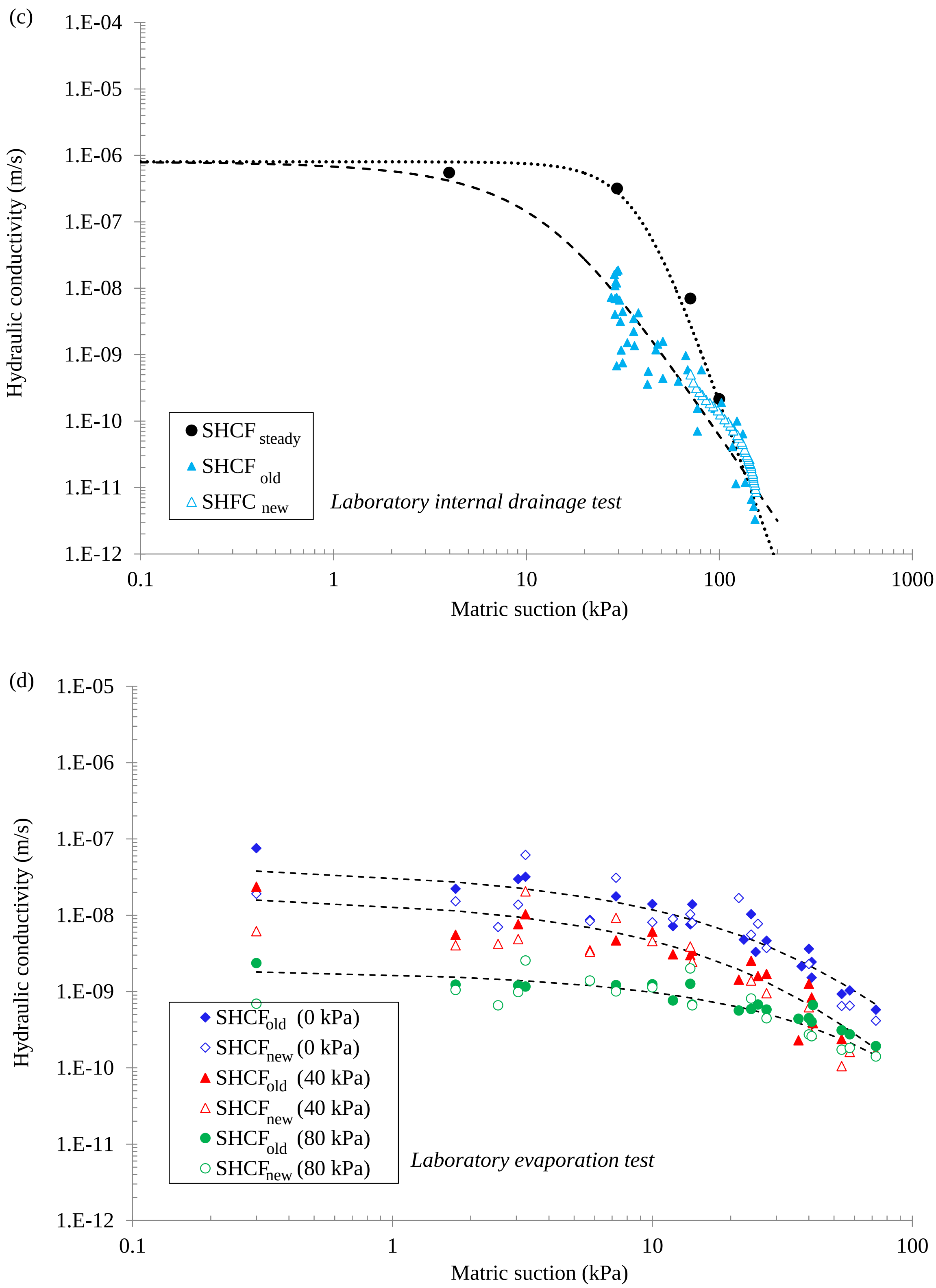
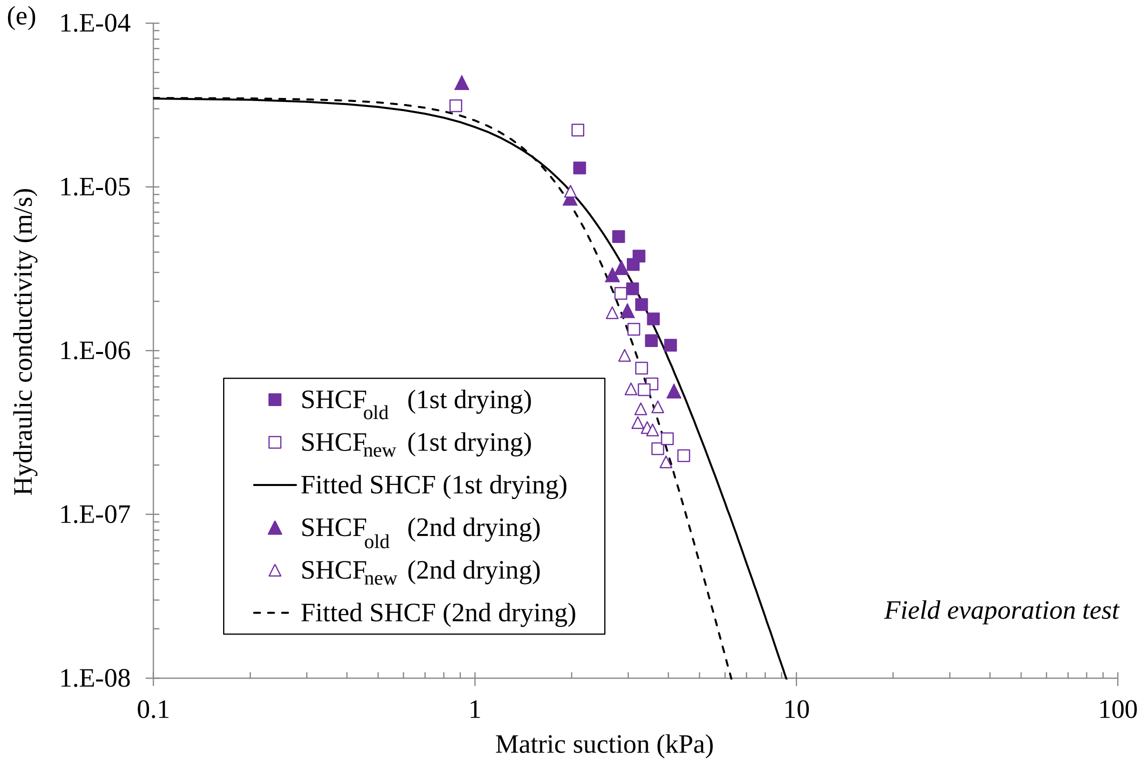


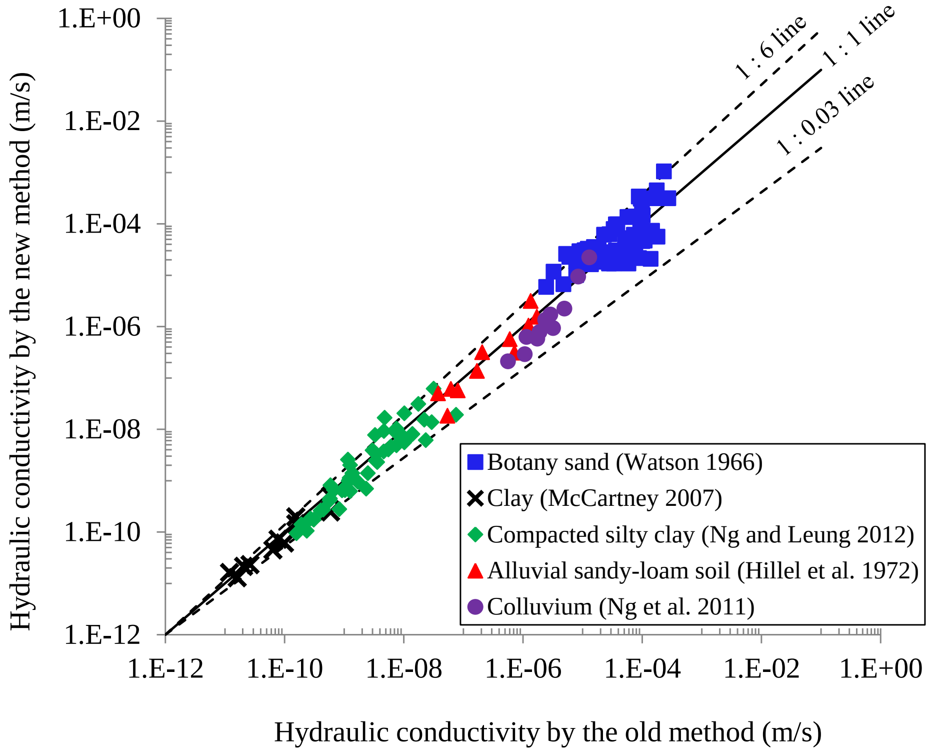
Figure 6. Comparisons of SHCF$_{old}$ and SHCF$_{new}$ of (a) Botany sand (Watson 1966); (b) alluvial sandy-loam soil (Hillel et al. 1972)



Con't Fig. 6. Comparisons of $SHCF_{old}$ and $SHCF_{new}$ of (c) clay (McCartney 2007); and (d) compacted silty clay (Ng and Leung 2012);



Con’t Fig. 6. Comparisons of $SHCF_{old}$ and $SHCF_{new}$ of (e) colluvium (Ng et al. 2011)



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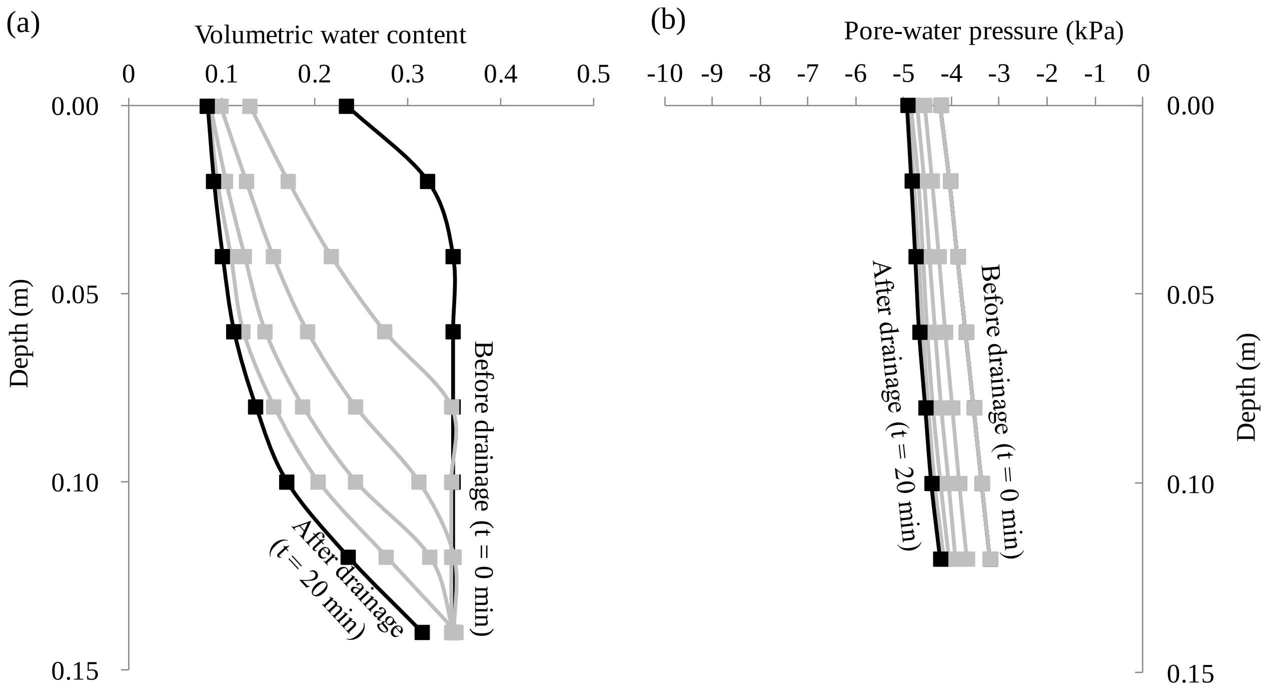
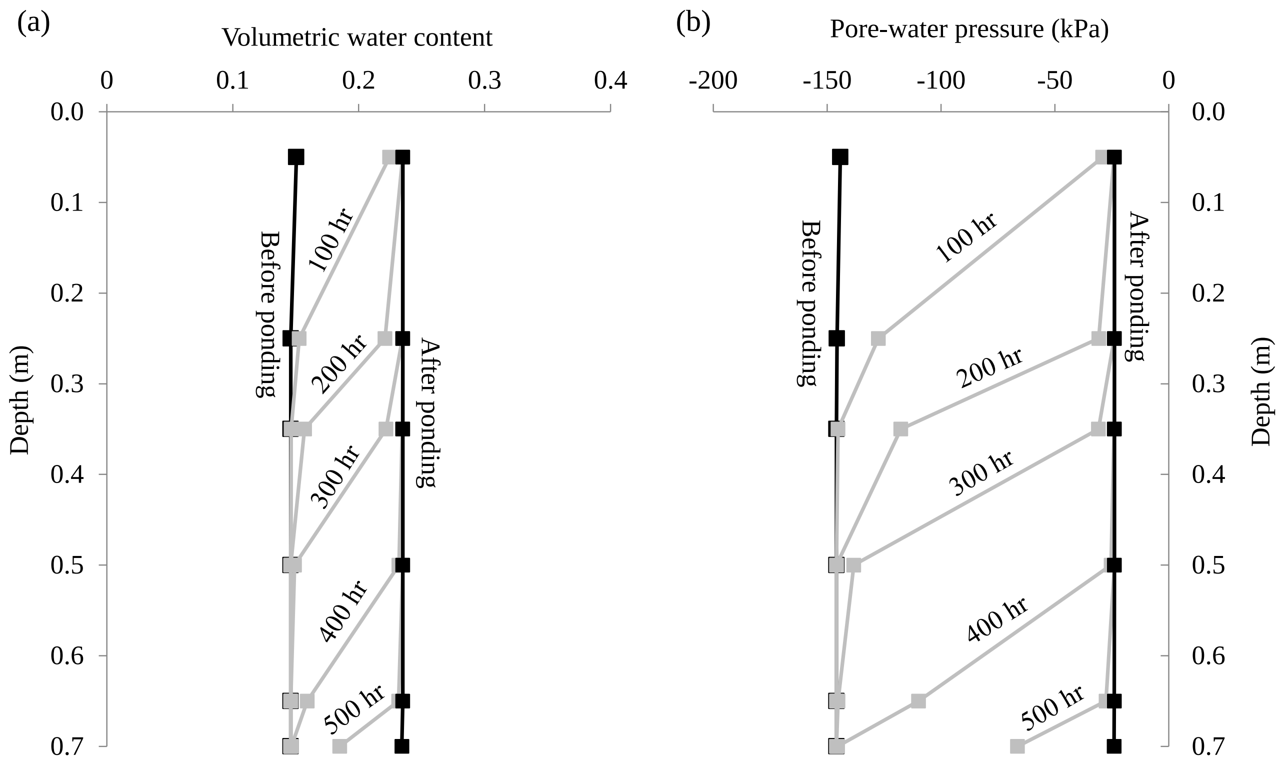


Figure A1. Measured instantaneous profiles of (a) VWC and (b) PWP (4 min isochrones) of in the top 0.15 m region of a 0.57 m-tall Botany sand column reported by Watson (1966). Note that the responses of soil below 0.15 m depth were not reported by the original study.



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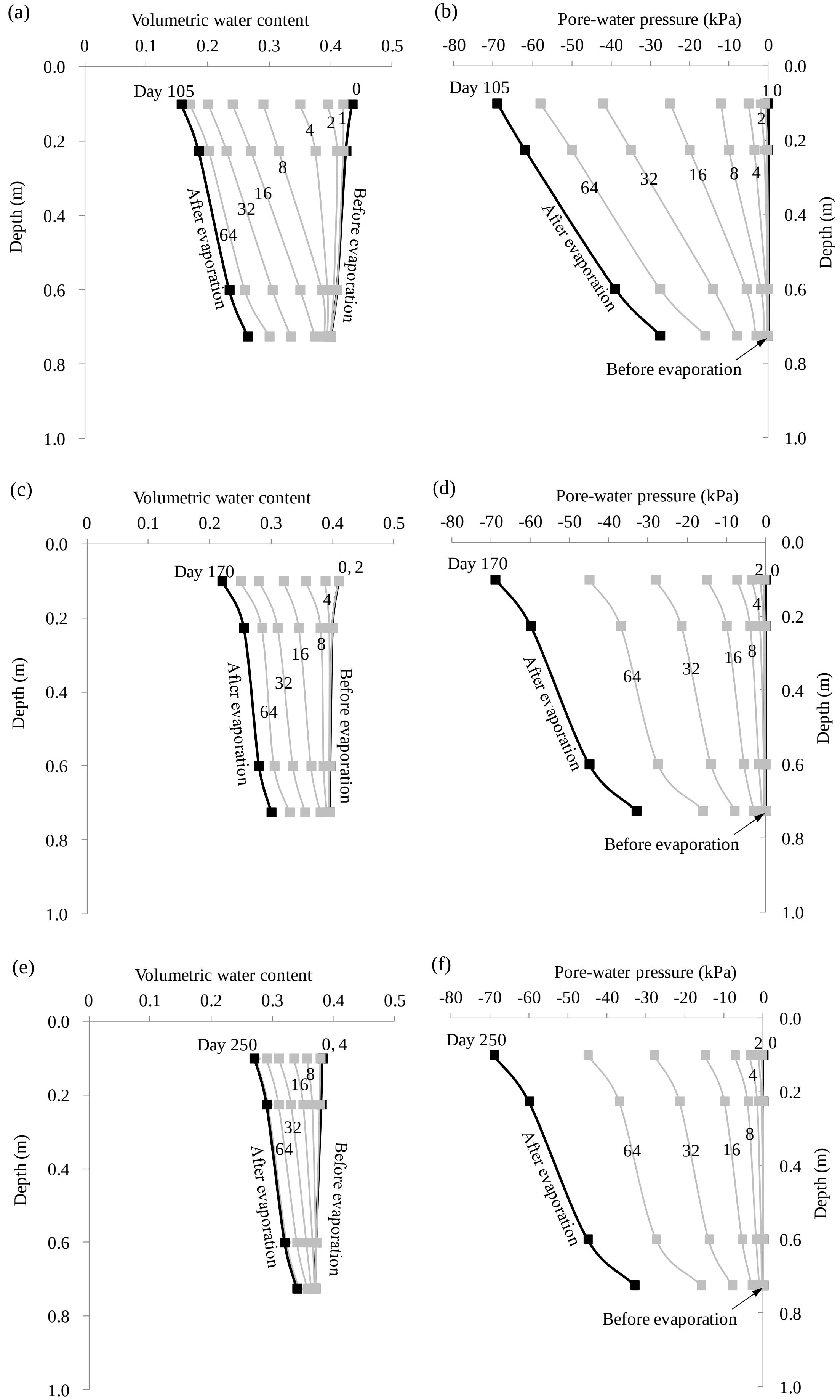

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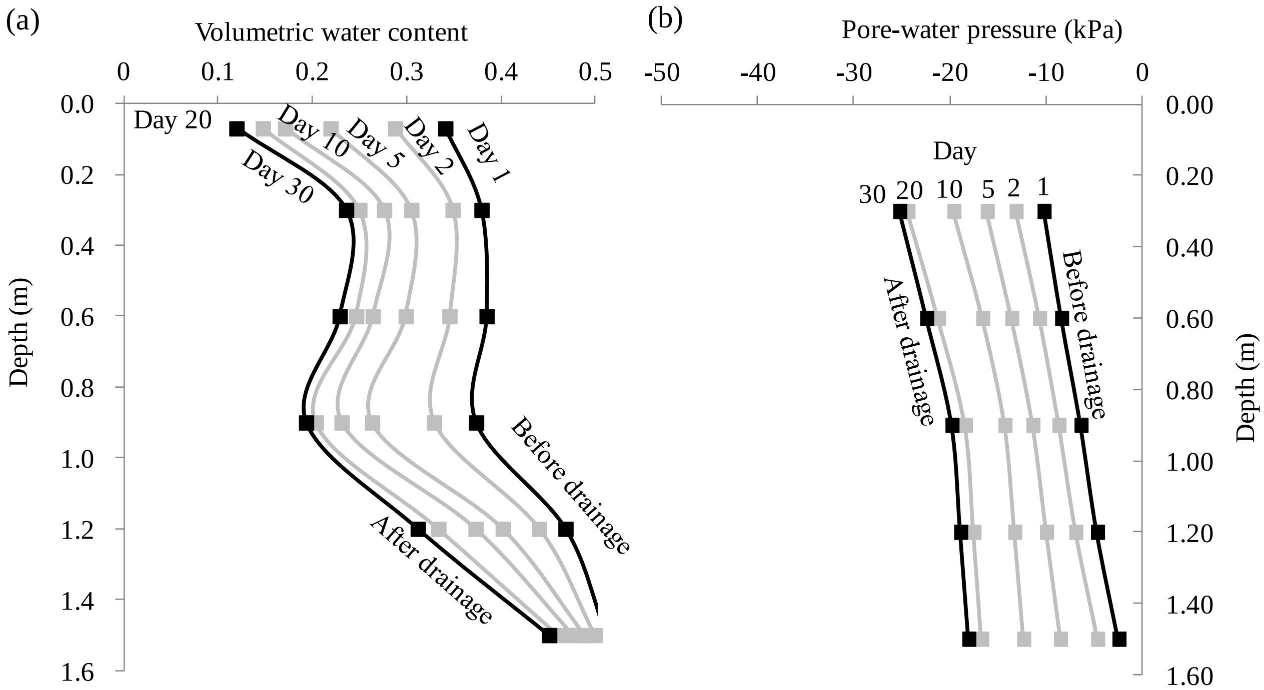

Figure A4. Measured instantaneous profiles of (a) VWC and (b) PWP of alluvial sandy-loam ground reported by Hillel et al. (1972)

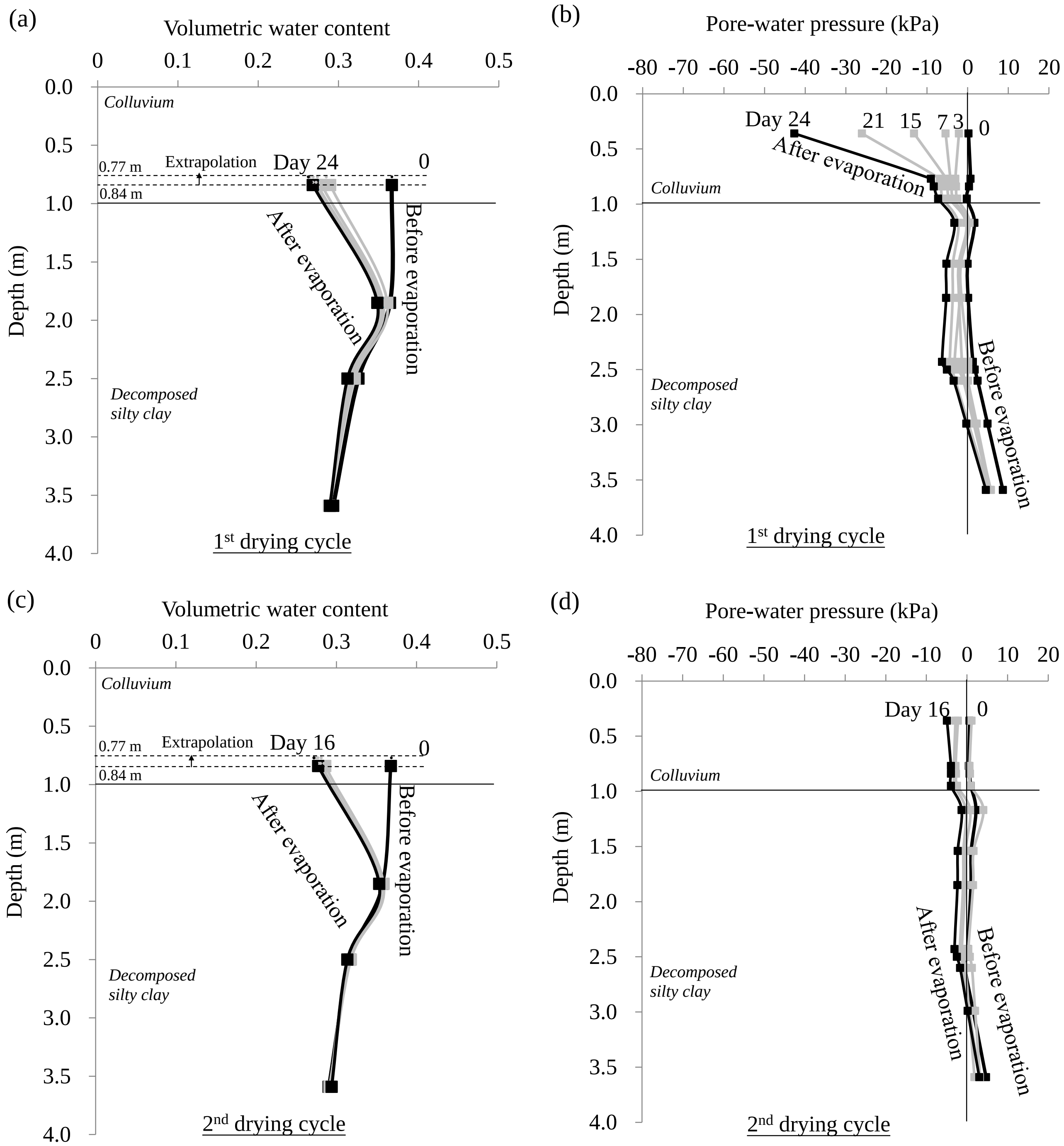

Figure A5. Measured instantaneous profiles of (a) VWC and (b) PWP of colluvium and decomposed silty clay during the 1st drying cycle; and (c) VWC and (d) PWP during the 2nd drying cycle reported by Ng et al. (2011)